Lambek Calculus Extended with Subexponential and Bracket Modalities

Max Kanovich, Stepan Kuznetsov, Andre Scedrov

Basic Categorial Grammar

John loves Mary

Basic Categorial Grammar

John	loves	Mary
np	$(np \backslash s) / np$	np

Basic Categorial Grammar

John	loves	Mary	
np	$(np \backslash s) / np$	np	$\to s$

Basic Categorial Grammar

	John	loves	Mary	
$\vdash$	np	$(np \backslash s) / np$	np	$\rightarrow s$

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Non-commutativity: $\vdash np, np \backslash s \to np$ ("John runs"), but $\not\vdash np \backslash s, np \to s)$ ("runs John").

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Reduction rules of BCG: $A, A \backslash B \rightarrow B$; $\qquad B / A, A \rightarrow B$

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[Ajdukiewicz 1935, Bar-Hillel *et al.* 1960]

Extending Categorial Grammar

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Extending Categorial Grammar

	John	loves	Mary	
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the girl whom John loves

Extending Categorial Grammar

	John	loves	Mary	
$\vdash$	np	$(np \backslash s) / np$	np	$\rightarrow s$

the	girl	whom	John	loves
np / n	n	$(n \backslash n) / (s / np)$	np	$(np \backslash s) / np$

Extending Categorial Grammar

$$\begin{array}{lllll} & \text{John} & \text{loves} & \text{Mary} & \\ \vdash & np & (np \backslash s) / np & np & \rightarrow s \end{array}$$

$$\begin{array}{lllll} \text{the} & \text{girl} & \text{whom} & \text{John} & \text{loves} \\ np / n & n & (n \backslash n) / (s / np) & \underbrace{np \quad (np \backslash s) / np}_{\rightarrow s / np} \end{array}$$

Extending Categorial Grammar

	John	loves	Mary	
$\vdash$	np	$(np \backslash s) / np$	np	$\rightarrow s$

	the	girl	whom	John	loves	
$\vdash$	np / n	n	$(n \backslash n) / (s / np)$	np	$(np \backslash s) / np$	$\rightarrow np$

$$\underbrace{np \quad (np \backslash s) / np}_{\rightarrow s / np}$$

Extending Categorial Grammar

	John	loves	Mary	
$\vdash$	np	$(np \backslash s) / np$	np	$\to s$

	the	girl	whom	John	loves	
$\vdash$	np / n	n	$(n \backslash n) / (s / np)$	np	$(np \backslash s) / np$	$\to np$

$$\underbrace{np \quad (np \backslash s) / np}_{\to s / np}$$

Deriving principles like $np, (np \backslash s) / np \to s / np$ requires extra rules (in this particular case, associativity: $(A \backslash B) / C \leftrightarrow A \backslash (B / C)$).

Extending Categorial Grammar

	John	loves	Mary	
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$\vdash$	np / n	n	$(n \backslash n) / (s / np)$	np	$(np \backslash s) / np$	$\rightarrow np$

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the	boy	who	loves	Mary

Extending Categorial Grammar

	John	loves	Mary	
$\vdash$	np	$(np \backslash s) / np$	np	$\rightarrow s$

	the	girl	whom	John	loves	
$\vdash$	np / n	n	$(n \backslash n) / (s / np)$	np	$(np \backslash s) / np$	$\rightarrow np$

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Deriving principles like $np, (np \backslash s) / np \rightarrow s / np$ requires extra rules (in this particular case, associativity: $(A \backslash B) / C \leftrightarrow A \backslash (B / C)$).

the	boy	who	loves	Mary
np / n	n	$(n \backslash n) / (np \backslash s)$	$(np \backslash s) / np$	np

Extending Categorial Grammar

$$\begin{array}{lccccc} & \text{John} & \text{loves} & \text{Mary} & \\ \vdash & np & (np \backslash s) / np & np & \rightarrow s \end{array}$$

$$\begin{array}{lcccccc} & \text{the} & \text{girl} & \text{whom} & \text{John} & \text{loves} & \\ \vdash & np / n & n & (n \backslash n) / (s / np) & \underbrace{np \quad (np \backslash s) / np}_{\rightarrow s / np} & & \rightarrow np \end{array}$$

Deriving principles like $np, (np \backslash s) / np \rightarrow s / np$ requires extra rules (in this particular case, associativity: $(A \backslash B) / C \leftrightarrow A \backslash (B / C)$).

$$\begin{array}{ccccc} \text{the} & \text{boy} & \text{who} & \text{loves} & \text{Mary} \\ np / n & n & (n \backslash n) / (np \backslash s) & \underbrace{(np \backslash s) / np \quad np}_{\rightarrow np \backslash s} & \end{array}$$

Extending Categorial Grammar

$$\vdash \quad \underset{\text{John}}{np} \quad \underset{\text{loves}}{(np \backslash s) / np} \quad \underset{\text{Mary}}{np} \quad \rightarrow s$$

$$\vdash \quad \underset{\text{the}}{np / n} \quad \underset{\text{girl}}{n} \quad \underset{\text{whom}_i}{(n \backslash n) / (s / np)} \quad \underbrace{\underset{\text{John}}{np} \quad \underset{\text{loves}}{(np \backslash s) / np}}_{\rightarrow s / np} \quad \underset{e_i}{} \rightarrow np$$

Deriving principles like $np, (np \backslash s) / np \rightarrow s / np$ requires extra rules (in this particular case, associativity: $(A \backslash B) / C \leftrightarrow A \backslash (B / C)$).

$$\vdash \quad \underset{\text{the}}{np / n} \quad \underset{\text{boy}}{n} \quad \underset{\text{who}}{(n \backslash n) / (np \backslash s)} \quad \underbrace{\underset{\text{loves}}{(np \backslash s) / np} \quad \underset{\text{Mary}}{np}}_{\rightarrow np \backslash s} \quad \rightarrow np$$

Extending Categorial Grammar

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	the	girl	whom$_i$	John	loves	e_i
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Deriving principles like $np, (np \backslash s) / np \rightarrow s / np$ requires extra rules (in this particular case, associativity: $(A \backslash B) / C \leftrightarrow A \backslash (B / C)$).

	the	boy	who$_i$	e_i	loves	Mary	
$\vdash$	np / n	n	$(n \backslash n) / (np \backslash s)$		$(np \backslash s) / np$	np	$\rightarrow np$

$$\underbrace{(np \backslash s) / np \quad np}_{\rightarrow np \backslash s}$$

Extending Categorial Grammar (cont.)

Another example (from Italian, see [Moot and Retoré 2012]):
"She/He watches the train passing"

Guarda	passare	il	treno
She/He watches	pass	the	train

Extending Categorial Grammar (cont.)

Another example (from Italian, see [Moot and Retoré 2012]):
"She/He watches the train passing"

	Guarda	passare	il	treno	
	She/He watches	pass	the	train	
$\vdash$	$s\ /\ inf$	$inf\ /\ np$	$np\ /\ n$	n	$\rightarrow s$

Extending Categorial Grammar (cont.)

Another example (from Italian, see [Moot and Retoré 2012]):
"She/He watches the train passing"

	Guarda	passare	il	treno	
	She/He watches	pass	the	train	
$\vdash$	$s \,/\, inf$	$inf \,/\, np$	$np \,/\, n$	n	$\rightarrow s$

Transform into a question: "What does she/he watch passing?"

Cosa	guarda	passare ?	
			$\rightarrow s$

Here we need transitivity: $A \,/\, B, B \,/\, C \rightarrow A \,/\, C$.

Extending Categorial Grammar (cont.)

Another example (from Italian, see [Moot and Retoré 2012]):
"She/He watches the train passing"

	Guarda	passare	il	treno	
	She/He watches	pass	the	train	
$\vdash$	$s \,/\, inf$	$inf \,/\, np$	$np \,/\, n$	n	$\rightarrow s$

Transform into a question: "What does she/he watch passing?"

	Cosa	guarda	passare ?	
$\vdash$	$q \,/(s \,/\, np)$	$s \,/\, inf$	$inf \,/\, np$	$\rightarrow s$

Here we need transitivity: $A \,/\, B, B \,/\, C \rightarrow A \,/\, C$.

Extending Categorial Grammar (cont.)

Another example (from Italian, see [Moot and Retoré 2012]): "She/He watches the train passing"

	Guarda	passare	il	treno	
	She/He watches	pass	the	train	
$\vdash$	$s \,/\, inf$	$inf \,/\, np$	$np \,/\, n$	n	$\rightarrow s$

Transform into a question: "What does she/he watch passing?"

$$\vdash \quad \underset{q\,/(s\,/\,np)}{\text{Cosa}} \quad \underbrace{\underset{s\,/\,inf}{\text{guarda}} \quad \underset{inf\,/\,np}{\text{passare ?}}}_{\rightarrow\, s\,/\,np} \quad \rightarrow s$$

Here we need transitivity: $A \,/\, B, B \,/\, C \rightarrow A \,/\, C$.

Extending Categorial Grammar: Two Approaches

1. Add necessary principles as extra axioms to BCG
   $\rightsquigarrow$ Combinatory Categorial Grammar (CCG) [Steedman 1996]

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1. Add necessary principles as extra axioms to BCG
   $\rightsquigarrow$ Combinatory Categorial Grammar (CCG) [Steedman 1996]
2. One calculus to derive them all! $\rightsquigarrow$ Lambek Grammar [Lambek 1958]

The Lambek Calculus ($\mathbf{L}^*$)

$$\overline{A \to A}$$

$$\frac{\Pi \to A \quad \Delta_1, B, \Delta_2 \to C}{\Delta_1, B \,/\, A, \Pi, \Delta_2 \to C}\ (/ \to) \qquad \frac{\Pi, A \to B}{\Pi \to B \,/\, A}\ (\to /)$$

$$\frac{\Pi \to A \quad \Delta_1, B, \Delta_2 \to C}{\Delta_1, \Pi, A \setminus B, \Delta_2 \to C}\ (\setminus \to) \qquad \frac{A, \Pi \to B}{\Pi \to A \setminus B}\ (\to \setminus)$$

[Lambek 1958, 1961, ...]

The Lambek Calculus ($\mathbf{L}^*$)

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$$\frac{\Pi \to A \quad \Delta_1, B, \Delta_2 \to C}{\Delta_1, B \,/\, A, \Pi, \Delta_2 \to C} \ (/ \to) \qquad \frac{\Pi, A \to B}{\Pi \to B \,/\, A} \ (\to /)$$

$$\frac{\Pi \to A \quad \Delta_1, B, \Delta_2 \to C}{\Delta_1, \Pi, A \setminus B, \Delta_2 \to C} \ (\setminus \to) \qquad \frac{A, \Pi \to B}{\Pi \to A \setminus B} \ (\to \setminus)$$

[Lambek 1958, 1961, ...]

$\mathbf{L}^* \vdash (A \setminus B) \,/\, C \leftrightarrow A \setminus (B \,/\, C)$

$\mathbf{L}^* \vdash A \,/\, B, B \,/\, C \to A \,/\, C$

$\ldots$

Properties of the Lambek Calculus

- Lambek grammars generate precisely context-free languages [Pentus 1993].
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- The Lambek calculus is NP-complete [Pentus 2006, Savateev 2008].
  (Steedman's CCGs enjoy polynomial-time parsing.)

Properties of the Lambek Calculus

- Lambek grammars generate precisely context-free languages [Pentus 1993].
  This means that formally their expressive power is not greater than the power of BCGs.
- The Lambek calculus is NP-complete [Pentus 2006, Savateev 2008].
  (Steedman's CCGs enjoy polynomial-time parsing.)
- Polynomial-time algorithm for fragments of bounded depth [Pentus 2010].
  (Running time $O(2^d n^4)$, where n is the length of the sequent and d is the implication nesting depth.)

Unwanted Derivations

book which John laughed without reading

Unwanted Derivations

book	which	John laughed without reading
CN	$(CN \setminus CN)/(S / N)$	$\underbrace{\hphantom{John laughed without reading}}_{S / N}$

Unwanted Derivations

$$\vdash \underset{CN}{\text{book}} \quad \underset{(CN \backslash CN) / (S / N)}{\text{which}} \quad \underbrace{\text{John laughed without reading}}_{S / N} \rightarrow CN$$

Unwanted Derivations

$$\vdash \underset{CN}{\text{book}} \quad \underset{(CN \backslash CN)/(S/N)}{\text{which}} \quad \underbrace{\text{John laughed without reading}}_{S/N} \rightarrow CN$$

Unwanted Derivations

$$
\begin{array}{ccccc}
* & \text{book} & \text{which} & \text{John laughed without reading} & \\
\vdash & CN & (CN \setminus CN)/(S / N) & \underbrace{\hphantom{\text{John laughed without reading}}}_{S / N} & \rightarrow CN
\end{array}
$$

Unwanted Derivations

*	book	which	John laughed without reading	
$\vdash$	CN	$(CN \setminus CN)/(S/N)$	$\underbrace{\hspace{6em}}_{S/N}$	$\to CN$

*	girl	who	John likes Mary and Pete likes

Unwanted Derivations

$$\begin{array}{llll} * & \text{book} & \text{which} & \text{John laughed without reading} \\ \vdash & CN & (CN \backslash CN)/(S/N) & \underbrace{\hspace{6cm}}_{S/N} \rightarrow CN \end{array}$$

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Unwanted Derivations

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$$\begin{array}{lcccc} * & \text{girl} & \text{who} & \text{John likes Mary and Pete likes} & \\ \vdash & CN & (CN \setminus CN) / (S / N) & \underbrace{\hphantom{\text{John likes Mary and Pete likes}}}_{S / N} & \rightarrow CN \end{array}$$

(cf. “John likes Mary and Pete likes Kate” $\rightarrow S$; “and” is of type $S \setminus S / S$)

The Lambek Calculus with Brackets

[Morrill 1992, Moortgat 1995]

$$\frac{}{A \to A}$$

$$\frac{\Pi \to A \quad \Delta(B) \to C}{\Delta(\Pi, A \setminus B) \to C} \qquad \frac{A, \Pi \to B}{\Pi \to A \setminus B} \qquad \frac{\Gamma(A, B) \to C}{\Gamma(A \cdot B) \to C}$$

$$\frac{\Pi \to A \quad \Delta(B) \to C}{\Delta(B \,/\, A, \Pi) \to C} \qquad \frac{\Pi, A \to B}{\Pi \to B \,/\, A} \qquad \frac{\Gamma \to A \quad \Delta \to B}{\Gamma, \Delta \to A \cdot B}$$

$$\frac{\Delta([A]) \to C}{\Delta(\langle\rangle A) \to C} \qquad \frac{\Pi \to A}{[\Pi] \to \langle\rangle A} \qquad \frac{\Delta(A) \to C}{\Delta([[]^{-1} A]) \to C} \qquad \frac{[\Pi] \to A}{\Pi \to []^{-1} A}$$

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- Brackets introduce *controlled non-associativity.*

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$$\frac{\Delta([A]) \to C}{\Delta(\langle\rangle A) \to C} \qquad \frac{\Pi \to A}{[\Pi] \to \langle\rangle A} \qquad \frac{\Delta(A) \to C}{\Delta([[]^{-1} A]) \to C} \qquad \frac{[\Pi] \to A}{\Pi \to []^{-1} A}$$

- Brackets introduce *controlled non-associativity.*
- Cut elimination proved by Moortgat [1996].

Islands: Blocking Unwanted Derivations Using Brackets

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- book which John laughed without reading

Islands: Blocking Unwanted Derivations Using Brackets

- book which John laughed [without reading]

Islands: Blocking Unwanted Derivations Using Brackets

- book which John laughed [without reading]

$$CN, (CN \setminus CN) / (S / CN), N, N \setminus S, [[]^{-1}((N \setminus S) \setminus (N \setminus S)) / (N \setminus S), (N \setminus S) / N] \rightarrow CN$$

Islands: Blocking Unwanted Derivations Using Brackets

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$CN, (CN \backslash CN) / (S / CN), N, N \backslash S, [[]^{-1}((N \backslash S) \backslash (N \backslash S)) / (N \backslash S), (N \backslash S) / N] \rightarrow CN$

This sequent is not derivable.

Islands: Blocking Unwanted Derivations Using Brackets

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  This sequent is not derivable.

- girl who John likes Mary and Pete likes

Islands: Blocking Unwanted Derivations Using Brackets

- book which John laughed [without reading]

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- girl who [John likes Mary and Pete likes]

Islands: Blocking Unwanted Derivations Using Brackets

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$CN, (CN \backslash CN) / (S / CN), N, N \backslash S, [[]^{-1}((N \backslash S) \backslash (N \backslash S)) / (N \backslash S), (N \backslash S) / N] \rightarrow CN$

  This sequent is not derivable.

- girl who [John likes Mary and Pete likes]

$CN, (CN \backslash CN) / (S / CN), [N, (N \backslash S) / N, N, (S \backslash []^{-1} S) / S, N, (N \backslash S) / N] \rightarrow CN$

Islands: Blocking Unwanted Derivations Using Brackets

- book which John laughed [without reading]

$CN, (CN \backslash CN) / (S / CN), N, N \backslash S, [[]^{-1}((N \backslash S) \backslash (N \backslash S)) / (N \backslash S), (N \backslash S) / N] \rightarrow CN$

  This sequent is not derivable.

- girl who [John likes Mary and Pete likes]

$CN, (CN \backslash CN) / (S / CN), [N, (N \backslash S) / N, N, (S \backslash []^{-1} S) / S, N, (N \backslash S) / N] \rightarrow CN$

  Neither is this one.

Subexponential: Medial Extraction

the girl whom John met yesterday

Subexponential: Medial Extraction

the girl whom$_i$ John met e_i yesterday

Subexponential: Medial Extraction

the girl whom$_i$ John met e_i yesterday

Subexponential: Medial Extraction

the girl whom$_i$ John met e_i yesterday

$$\underbrace{\text{John met } e_i \text{ yesterday}}_{\to\, S \,/\, !N}$$

$$\frac{\Delta(!A, \Gamma) \to C}{\Delta(\Gamma, !A) \to C}\ (\mathrm{perm}_1) \qquad \frac{\Delta(A) \to C}{\Delta(!A) \to C}\ (! \to)$$

Subexponential: Medial Extraction

the girl whom$_i$ $\underbrace{\text{John met } e_i \text{ yesterday}}_{\to S \,/\, !N}$

$$\frac{\Delta(!A, \Gamma) \to C}{\Delta(\Gamma, !A) \to C}\ (\mathrm{perm}_1) \qquad \frac{\Delta(A) \to C}{\Delta(!A) \to C}\ (! \to)$$

$$\cfrac{\cfrac{\cfrac{N, (N \setminus S) / N, N, (N \setminus S) \setminus (N \setminus S) \to S}{N, (N \setminus S) / N, !N, (N \setminus S) \setminus (N \setminus S) \to S}\ (! \to)}{N, (N \setminus S) / N, (N \setminus S) \setminus (N \setminus S), !N \to S}\ (\mathrm{perm}_1)}{N, (N \setminus S) / N, (N \setminus S) \setminus (N \setminus S) \to S \,/\, !N}\ (\to /)$$

Subexponential: Medial Extraction

the girl — whom$_i$ $(CN \setminus CN) / (S \,/\, !N)$ — $\underbrace{\text{John met } e_i \text{ yesterday}}_{\to\, S \,/\, !N}$

$$\frac{\Delta(!A, \Gamma) \to C}{\Delta(\Gamma, !A) \to C}\ (\mathrm{perm}_1) \qquad \frac{\Delta(A) \to C}{\Delta(!A) \to C}\ (! \to)$$

$$\dfrac{\dfrac{\dfrac{N, (N \setminus S) / N, N, (N \setminus S) \setminus (N \setminus S) \to S}{N, (N \setminus S) / N, !N, (N \setminus S) \setminus (N \setminus S) \to S}\ (! \to)}{N, (N \setminus S) / N, (N \setminus S) \setminus (N \setminus S), !N \to S}\ (\mathrm{perm}_1)}{N, (N \setminus S) / N, (N \setminus S) \setminus (N \setminus S) \to S \,/\, !N}\ (\to /)$$

Subexponential: Medial Extraction

the girl	whom$_i$	John met e_i yesterday	
$\ldots$	$(CN \setminus CN) / (S / !N)$	$\underbrace{\qquad\qquad}_{\rightarrow S / !N}$	$\rightarrow N$

$$\frac{\Delta(!A, \Gamma) \rightarrow C}{\Delta(\Gamma, !A) \rightarrow C}\ (\mathrm{perm}_1) \qquad \frac{\Delta(A) \rightarrow C}{\Delta(!A) \rightarrow C}\ (! \rightarrow)$$

$$\cfrac{\cfrac{\cfrac{N, (N \setminus S) / N, N, (N \setminus S) \setminus (N \setminus S) \rightarrow S}{N, (N \setminus S) / N, !N, (N \setminus S) \setminus (N \setminus S) \rightarrow S}\ (! \rightarrow)}{N, (N \setminus S) / N, (N \setminus S) \setminus (N \setminus S), !N \rightarrow S}\ (\mathrm{perm}_1)}{N, (N \setminus S) / N, (N \setminus S) \setminus (N \setminus S) \rightarrow S / !N}\ (\rightarrow /)$$

Subexponential: Parasitic Extraction

the paper that John signed without reading

Subexponential: Parasitic Extraction

the paper that$_i$ John signed e_i without reading e_i

Subexponential: Parasitic Extraction

$$\text{the paper} \quad \text{that}_i \quad \underbrace{\text{John signed } e_i \text{ without reading } e_i}_{\rightarrow\, S\, /\, !N}$$

Subexponential: Parasitic Extraction

the paper that$_i$ $\underbrace{\text{John signed } e_i \text{ without reading } e_i}_{\to S \,/\, !N}$

$$\frac{\Delta(!A, \Gamma) \to C}{\Delta(\Gamma, !A) \to C}\ (\mathrm{perm}_1) \qquad \frac{\Delta(A) \to C}{\Delta(!A) \to C}\ (! \to)$$

$$\frac{\Delta(!A, !A) \to C}{\Delta(!A) \to C}\ (\mathrm{contr})$$

Subexponential: Parasitic Extraction

the paper that$_i$ $\underbrace{\text{John signed } e_i \text{ [without reading } e_i]}_{\to S \,/\, !N}$

$$\frac{\Delta(!A, \Gamma) \to C}{\Delta(\Gamma, !A) \to C}\ (\mathrm{perm}_1) \qquad \frac{\Delta(A) \to C}{\Delta(!A) \to C}\ (! \to)$$

$$\frac{\Delta(!A, !A) \to C}{\Delta(!A) \to C}\ (\mathrm{contr})$$

Subexponential: Parasitic Extraction

$$\text{the paper} \quad \text{that}_i \quad \underbrace{\text{John signed } e_i \text{ [without reading } e_i\text{]}}_{\to S \,/\, !N}$$

$$\frac{\Delta(!A, \Gamma) \to C}{\Delta(\Gamma, !A) \to C} \text{ (perm}_1\text{)} \qquad \frac{\Delta(A) \to C}{\Delta(!A) \to C} \text{ (! }\to\text{)}$$

$$\frac{\Delta(!A_1, \ldots, !A_n, [!A_1, \ldots, !A_n, \Gamma]) \to B}{\Delta(!A_1, \ldots, !A_n, \Gamma) \to B} \text{ (contr}_{\mathbf{b}}\text{)}$$

Subexponential: Parasitic Extraction

the paper that$_i$ $\underbrace{\text{John signed } e_i \text{ [without reading } e_i]}_{\to S \,/\, !N}$

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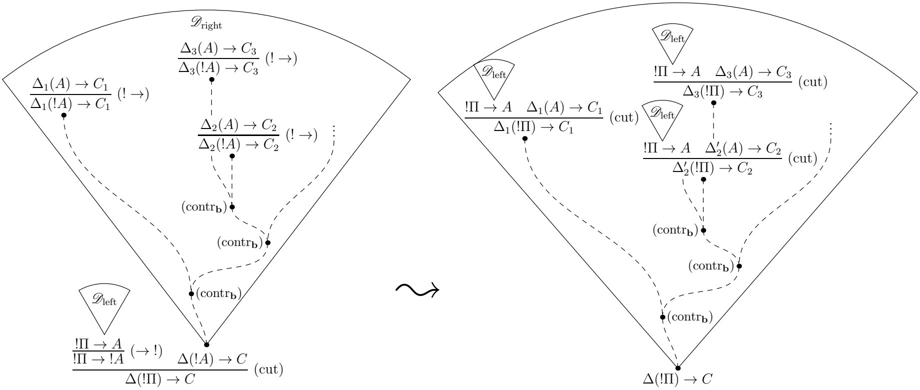

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$$!\Gamma = !B_1, \ldots, !B_n,$$

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$$!\Phi = !(\mathbf{1}\,/(!B_1)), \ldots, !(\mathbf{1}\,/(!B_n)), \text{ and}$$

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4. $s \Rightarrow^* a_1 \ldots a_n$ *in the type-0 grammar.*

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Thank you !